COURSE: AP Calculus BC	GRADE: 12
UNIT 1: Functions and Graphs	TIME FRAME: 7 Days

PA ACADEMIC STANDARDS FOR MATHEMATICS:

M11.A.1	**Demonstrate an understanding of numbers, ways of representing numbers, relationships among numbers and number systems.**
M11.A.1.1	Represent and/or use numbers in equivalent forms (e.g., integers, fractions, decimals, percents, square roots, exponents and scientific notation).
M11.A.1.1.1	Find the square root of an integer to the nearest tenth using either a calculator or estimation.
M11.A.1.1.2	Express numbers and/or simplify expressions using scientific notation (including numbers less than 1).
M11.A.1.1.3	Simplify square roots.
M11.A.2	**Understand the meanings of operations, use operations and understand how they relate to each other.**
M11.A.2.1	Apply ratio and/or proportion in problem-solving situations.
M11.A.2.1.1	Solve problems using operations with rational numbers including rates and percents (single and multi-step and multiple procedure operations) (e.g., distance, work and mixture problems, etc.).
M11.A.2.1.2	Solve problems using direct and inverse proportions.
M11.A.2.1.3	Identify and/or use proportional relationships in problem solving settings.
M11.A.2.2	Use exponents, roots and/or absolute value to solve problems.
M11.A.2.2.1	Simplify/evaluate expressions involving positive and negative exponents, roots and/or absolute value (may contain all types of real numbers - exponents should not exceed power of 10).
M11.A.2.2.2	Simplify/evaluate expressions involving multiplying with exponents (e.g. $x^6 * x^7 = x^{13}$), powers of powers (e.g., $(x^6)^7=x^{42}$) and powers of products $(2x^2)^3=8x^6$.
M11.B.2	**Apply appropriate techniques, tools and formulas to determine measurements.**
M11.B.2.1	Use and/or compare measurements of angles.
M11.B.2.1.1	Measure and/or compare angles in degrees (up to 360°).
M11.C.1	**Analyze characteristics and properties of two- and three- dimensional geometric shapes and demonstrate understanding of geometric relationships.**
M11.C.1.4	Solve problems involving right triangles using the Pythagorean Theorem.
M11.C.1.4.1	Find the measure of a side of a right triangle using the Pythagorean Theorem.
M11.D.1	**Demonstrate an understanding of patterns, relations and functions.**
M11.D.1.1	Analyze and/or use patterns or relations.
M11.D.1.1.1	Analyze a set of data for the existence of a pattern and represent the pattern algebraically and/or graphically.
M11.D.1.1.2	Determine if a relation is a function given a set of points or a graph.
M11.D.1.1.3	Identify the domain, range or inverse of a relation (may be presented as ordered pairs or a table).
M11.D.2	**Represent and/or analyze mathematical situations using numbers, symbols, words, tables and/or graphs.**
M11.D.2.1	Write, solve and/or graph linear equations and inequalities using various methods.
M11.D.2.1.1	Solve compound inequalities and/or graph their solution sets on a number line.
M11.D.2.1.2	Identify or graph functions, linear equations or linear inequalities on a coordinate plane.
M11.D.2.1.3	Write, solve and/or apply a linear equation (including problem situations).
M11.D.2.1.4	Write and/or solve systems of equations using graphing, substitution and/or elimination
M11.D.2.1.5	Solve quadratic equations using factoring.

M11.D.2.2	Simplify expressions involving polynomials.
M11.D.2.2.1	Add, subtract and/or multiply polynomial expressions.
M11.D.2.2.2	Factor algebraic expressions, including difference of squares and trinomials.
M11.D.2.2.3	Simplify algebraic fractions.
M11.D.3	**Analyze change in various contexts.**
M11.D.3.1	Describe and/or determine change.
M11.D.3.1.1	Identify, describe and/or use constant or varying rates of change.
M11.D.3.1.2	Determine how a change in one variable relates to a change in a second variable.
M11.D.3.2	Compute and/or use the slope of a line.
M11.D.3.2.1	Apply the formula for the slope of a line to solve problems.
M11.D.3.2.2	Given the graph of the line, 2 points on the line, or the slope and a point on a line, write or identify the linear equation in point-slope, standard and/or slope-intercept form.
M11.D.3.2.3	Compute the slope and/or y-intercept represented by a linear equation or graph.

NCTM STANDARDS:

Numbers and Operations
- Understand numbers, ways of representing numbers, relationships among numbers, and number systems.
- Understand meanings of operations and how they relate to one another.
- Compute fluently and make reasonable estimates.

Algebra
- Understand patterns, relations, and functions.
- Represent and analyze mathematical situations and structures using algebraic symbols.
- Use mathematical models to represent and understand quantitative relationships.
- Analyze change in various contexts.

Geometry
- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
- Use visualization, spatial reasoning, and geometric modeling to solve problems.

Measurement
- Understand measurable attributes of objects and the units, systems, and processes of measurement.
- Apply appropriate techniques, tools, and formulas to determine measurements.

Problem Solving
- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.

UNIT OBJECTIVES:

1.1 Use previously learned mathematical skills and concepts to analyze and algebraically confirm the facts and completeness of a graph that is determined technologically. Predict behavior that is hidden from view on the calculator screen.

1.2 Use technology as a tool to investigate mathematical concepts and ideas to provide support for analytical work and to solve problems using approximation when analytic methods either fail or are impractical.

1.3 Use previously learned information about relations, functions, their graphs and transformations, their inverses, and their properties and formulas.

1.4 Determine local and global behavior of functions considered numerically, analytically, and by utilizing the support of graphing technology.

1.5 Model real-world phenomena.

• Monitor and reflect on the process of mathematical problem solving. **Reasoning and Proof** • Recognize reasoning and proof as fundamental aspects of mathematics. • Make and investigate mathematical conjectures. • Develop and evaluate mathematical arguments and proofs. • Select and use various types of reasoning and methods of proof. **Communication** • Organize and consolidate their mathematical thinking through communication. • Communicate their mathematical thinking coherently and clearly to peers, teachers, and others. • Analyze and evaluate the mathematical thinking and strategies of others. • Use the language of mathematics to express mathematical ideas precisely. **Connections** • Recognize and use connections among mathematical ideas. • Understand how mathematical ideas interconnect and build on one another to produce a coherent whole. • Recognize and apply mathematics in contexts outside of mathematics. **Representation** • Create and use representations to organize, record, and communicate mathematical ideas. • Select, apply, and translate among mathematical representations to solve problems. • Use representations to model and interpret physical, social, and mathematical phenomena.	

ACTIVITIES:

Constructively participate in individual and cooperative group work to explore, investigate, conjecture, discover, read, write, paraphrase, visualize, verbalize, and complete mathematical tasks.

Seek assistance from the instructor or other facilitators whenever necessary.

Use a graphing utility to draw graphs and determine a viewing window that gives the best complete graph.

Sketch graphs analytically.

Determine domain, range, intercepts, maximums, minimums, increasing and decreasing intervals, inflection points, and concavity of relations and functions.

Use the vertical and horizontal line tests to determine whether a relation is a function and whether the inverse is a function.

Find slope and equations for lines using various formulas.

Use the definition of absolute value and its properties.

Graph the absolute value function and other piecewise Functions.

Use the distance formula.

Test analytically for symmetry.

Test analytically to determine whether a function is even, odd, or neither.

Graph the greatest integer function.

Use the algebraic properties of functions.

Use the geometric transformations of shifting, reflecting, stretching and shrinking to analytically graph variations of known graphs.

Write equations for geometrically transformed graphs.

Use the general equation of a circle.

Determine inverse relations and inverse functions (compositional) graphically.

Test for one-to-one.

Determine inverses of linear, radical, rational, exponential and transcendental functions analytically.

Use the properties of logarithms.

Graph exponentials and logarithmic functions.

ASSESSMENTS:

Homework
Quizzes
Tests
Projects

REMEDIATION:

Precalculus Mathematics: A Graphing Approach , 2nd ed., Demana et. al. Resource Manual

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Calculus from Graphical, Numerical, and Symbolic Points of View. Ostebee and Zorn.

Lecture Guide and Student Notes

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Logarithm Combination Rules

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Adding Vectors Graphically (BC unit 9 only)

Vector Addition and Scalar Multiplication (BC unit 9 only)

Rectangular to Polar Components, Standard Position and Norm (BC unit 9 only)

ENRICHMENT:

AP Resources
www.apcentral.collegeboard.com

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Use radian and degree measure and their conversions. Find arc length. Graph the trigonometric functions and variations thereof and determine their respective domain, range, period, amplitude, and asymptotes (whenever applicable). Use the right triangle relationships. Graph the inverse trigonometric functions and variations. Use the trigonometric identities. Solve equations and inequalities. Solve various problems that model real-world Phenomena.	**RESOURCES:** Calculus: Graphical, Numerical, Algebraic 2007 Finney, Demana, Waits, Kennedy Barton, Brunsting, Diehl, Hill, Tyler, Wilson Pearson Education AP Test Prep Series: AP Calculus (student workbook for Finney text) Pearson Education. Boston, Massachusetts, 2007. TI-84 Plus Instructional Manual

Revised 8/08

COURSE: AP Calculus BC	GRADE: 12
UNIT 2: Limits and Continuity	TIME FRAME: 10 Days

PA ACADEMIC STANDARDS FOR MATHEMATICS:

M11.A.1	**Demonstrate an understanding of numbers, ways of representing numbers, relationships among numbers and number systems.**
M11.A.1.1	Represent and/or use numbers in equivalent forms (e.g., integers, fractions, decimals, percents, square roots, exponents and scientific notation).
M11.A.1.1.1	Find the square root of an integer to the nearest tenth using either a calculator or estimation.
M11.A.1.1.2	Express numbers and/or simplify expressions using scientific notation (including numbers less than 1).
M11.A.1.1.3	Simplify square roots.
M11.A.2	**Understand the meanings of operations, use operations and understand how they relate to each other.**
M11.A.2.1	Apply ratio and/or proportion in problem-solving situations.
M11.A.2.1.1	Solve problems using operations with rational numbers including rates and percents (single and multi-step and multiple procedure operations) (e.g., distance, work and mixture problems, etc.).
M11.A.2.1.2	Solve problems using direct and inverse proportions.
M11.A.2.1.3	Identify and/or use proportional relationships in problem solving settings.
M11.A.2.2	Use exponents, roots and/or absolute value to solve problems.
M11.A.2.2.1	Simplify/evaluate expressions involving positive and negative exponents, roots and/or absolute value (may contain all types of real numbers - exponents should not exceed power of 10).
M11.A.2.2.2	Simplify/evaluate expressions involving multiplying with exponents (e.g. $x^6 * x^7 = x^{13}$), powers of powers (e.g., $(x^6)^7=x^{42}$) and powers of products $(2x^2)^3=8x^6$.
M11.B.2	**Apply appropriate techniques, tools and formulas to determine measurements.**
M11.B.2.1	Use and/or compare measurements of angles.
M11.B.2.1.1	Measure and/or compare angles in degrees (up to 360°).
M11.C.1	**Analyze characteristics and properties of two- and three- dimensional geometric shapes and demonstrate understanding of geometric relationships.**
M11.C.1.4	Solve problems involving right triangles using the Pythagorean Theorem.
M11.C.1.4.1	Find the measure of a side of a right triangle using the Pythagorean Theorem.
M11.D.1	**Demonstrate an understanding of patterns, relations and functions.**
M11.D.1.1	Analyze and/or use patterns or relations.
M11.D.1.1.1	Analyze a set of data for the existence of a pattern and represent the pattern algebraically and/or graphically.
M11.D.1.1.2	Determine if a relation is a function given a set of points or a graph.
M11.D.1.1.3	Identify the domain, range or inverse of a relation (may be presented as ordered pairs or a table).
M11.D.2	**Represent and/or analyze mathematical situations using numbers, symbols, words, tables and/or graphs.**
M11.D.2.1	Write, solve and/or graph linear equations and inequalities using various methods.
M11.D.2.1.1	Solve compound inequalities and/or graph their solution sets on a number line.
M11.D.2.1.2	Identify or graph functions, linear equations or linear inequalities on a coordinate plane.
M11.D.2.1.3	Write, solve and/or apply a linear equation (including problem situations).
M11.D.2.1.4	Write and/or solve systems of equations using graphing, substitution and/or elimination.
M11.D.2.1.5	Solve quadratic equations using factoring.
M11.D.2.2	Simplify expressions involving polynomials.

M11.D.2.2.1	Add, subtract and/or multiply polynomial expressions.
M11.D.2.2.2	Factor algebraic expressions, including difference of squares and trinomials.
M11.D.2.2.3	Simplify algebraic fractions.
M11.D.3	**Analyze change in various contexts.**
M11.D.3.1	Describe and/or determine change.
M11.D.3.1.1	Identify, describe and/or use constant or varying rates of change.
M11.D.3.1.2	Determine how a change in one variable relates to a change in a second variable.
M11.D.3.2	Compute and/or use the slope of a line.
M11.D.3.2.1	Apply the formula for the slope of a line to solve problems.
M11.D.3.2.2	Given the graph of the line, 2 points on the line, or the slope and a point on a line, write or identify the linear equation in point-slope, standard and/or slope-intercept form.
M11.D.3.2.3	Compute the slope and/or y-intercept represented by a linear equation or graph.

NCTM STANDARDS:	**UNIT OBJECTIVES:**
Numbers and Operations • Understand numbers, ways of representing numbers, relationships among numbers, and number systems. • Understand meanings of operations and how they relate to one another. • Compute fluently and make reasonable estimates. **Algebra** • Understand patterns, relations, and functions. • Represent and analyze mathematical situations and structures using algebraic symbols. • Use mathematical models to represent and understand quantitative relationships. • Analyze change in various contexts. **Geometry** • Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships. • Use visualization, spatial reasoning, and geometric modeling to solve problems. **Measurement** • Understand measurable attributes of objects and the units, systems, and processes of measurement. • Apply appropriate techniques, tools, and formulas to determine measurements. **Problem Solving** • Build new mathematical knowledge through problem solving. • Solve problems that arise in mathematics and in other contexts. • Apply and adapt a variety of appropriate strategies to solve problems. • Monitor and reflect on the process of mathematical	2.1 Intuitively understand the notion of limit. Use it to help explain and predict observed local and global behavior of a function; and, realize that the calculus is built on the idea (concept) of limit. 2.2 Intuitively understand that the central idea of continuity is correctives and use its definition to analytically confirm continuity at a point. 2.3 Model real-world phenomena. 2.4 Connect and use previously learned skills and concepts with new analytical methods to confirm data suggested by technology. Use technology to support all analytical and numerical facts.

problem solving. **Reasoning and Proof** • Recognize reasoning and proof as fundamental aspects of mathematics. • Make and investigate mathematical conjectures. • Develop and evaluate mathematical arguments and proofs. • Select and use various types of reasoning and methods of proof. **Communication** • Organize and consolidate their mathematical thinking through communication. • Communicate their mathematical thinking coherently and clearly to peers, teachers, and others. • Analyze and evaluate the mathematical thinking and strategies of others. • Use the language of mathematics to express mathematical ideas precisely. **Connections** • Recognize and use connections among mathematical ideas. • Understand how mathematical ideas interconnect and build on one another to produce a coherent whole. • Recognize and apply mathematics in contexts outside of mathematics. **Representation** • Create and use representations to organize, record, and communicate mathematical ideas. • Select, apply, and translate among mathematical representations to solve problems. • Use representations to model and interpret physical, social, and mathematical phenomena.	

ACTIVITIES:

Constructively participate in individual and cooperative group work to explore, investigate, conjecture, discover, read, write, paraphrase, visualize, verbalize and complete mathematical tasks.

Seek assistance from the instructor and other facilitators whenever necessary.

Discuss and construct graphs and algebraic functions that have and do not have limits.

Paraphrase and use the limit properties.

Analytically confirm observed local and end behavior of a computer graph and predict hidden behavior.

Determine one-sided and two-sided limits.

Discuss and construct graphs and algebraic functions that are and are not continuous.

Paraphrase and use the analytic test to determine continuity at a point.

Paraphrase the concept of local linearity and construct initial concept of tangent lines.

Use the algebraic properties of continuous functions and redefine functions that have a removable discontinuity.

Paraphrase and use the max-min-and Intermediate Value Theorems for continuous functions.

Use the sandwich theorem to verify $\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$ (key fact in the analytic derivation of derivatives of trigonometric functions).

Determine finite limits as ∞.

Determine the nature of a graphs end behavior when $\lim_{x \to \infty} f(x)$ does not exist.

Determine the nature of the graphs local behavior when $\lim_{x \to \infty} f(x)$ does not exist.

Determine equations of vertical and horizontal asymptotes.

Model real-world phenomena of compound interest applications, natural behavior and connectivity.

ASSESSMENTS:

- Observation and questioning
- Discussions
- Projects and investigations
- Mathematical writing
- Homework
- Quizzes and Tests

REMEDIATION:

Precalculus Mathematics: A Graphing Approach , 2nd ed., Demana et. al. Resource Manual

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Revised 8/08

COURSE: AP Calculus BC	GRADE: 12
UNIT 3: Derivatives	TIME FRAME: 20 Days

PA ACADEMIC STANDARDS FOR MATHEMATICS:

M11.A.1	**Demonstrate an understanding of numbers, ways of representing numbers, relationships among numbers and number systems.**
M11.A.1.1	Represent and/or use numbers in equivalent forms (e.g., integers, fractions, decimals, percents, square roots, exponents and scientific notation).
M11.A.1.1.1	Find the square root of an integer to the nearest tenth using either a calculator or estimation.
M11.A.1.1.2	Express numbers and/or simplify expressions using scientific notation (including numbers less than 1).
M11.A.1.1.3	Simplify square roots.
M11.A.2	**Understand the meanings of operations, use operations and understand how they relate to each other.**
M11.A.2.1	Apply ratio and/or proportion in problem-solving situations.
M11.A.2.1.1	Solve problems using operations with rational numbers including rates and percents (single and multi-step and multiple procedure operations) (e.g., distance, work and mixture problems, etc.).
M11.A.2.1.2	Solve problems using direct and inverse proportions.
M11.A.2.1.3	Identify and/or use proportional relationships in problem solving settings.
M11.A.2.2	Use exponents, roots and/or absolute value to solve problems.
M11.A.2.2.1	Simplify/evaluate expressions involving positive and negative exponents, roots and/or absolute value (may contain all types of real numbers - exponents should not exceed power of 10).
M11.A.2.2.2	Simplify/evaluate expressions involving multiplying with exponents (e.g. $x^6 * x^7 = x^{13}$), powers of powers (e.g., $(x^6)^7=x^{42}$) and powers of products $(2x^2)^3=8x^6$.
M11.B.2	**Apply appropriate techniques, tools and formulas to determine measurements.**
M11.B.2.1	Use and/or compare measurements of angles.
M11.B.2.1.1	Measure and/or compare angles in degrees (up to 360°).
M11.C.1	**Analyze characteristics and properties of two- and three- dimensional geometric shapes and demonstrate understanding of geometric relationships.**
M11.C.1.4	Solve problems involving right triangles using the Pythagorean Theorem.
M11.C.1.4.1	Find the measure of a side of a right triangle using the Pythagorean Theorem.
M11.D.1	**Demonstrate an understanding of patterns, relations and functions.**
M11.D.1.1	Analyze and/or use patterns or relations.
M11.D.1.1.1	Analyze a set of data for the existence of a pattern and represent the pattern algebraically and/or graphically.
M11.D.1.1.2	Determine if a relation is a function given a set of points or a graph.
M11.D.1.1.3	Identify the domain, range or inverse of a relation (may be presented as ordered pairs or a table).
M11.D.2	**Represent and/or analyze mathematical situations using numbers, symbols, words, tables and/or graphs.**
M11.D.2.1	Write, solve and/or graph linear equations and inequalities using various methods.
M11.D.2.1.1	Solve compound inequalities and/or graph their solution sets on a number line.
M11.D.2.1.2	Identify or graph functions, linear equations or linear inequalities on a coordinate plane.
M11.D.2.1.3	Write, solve and/or apply a linear equation (including problem situations).
M11.D.2.1.4	Write and/or solve systems of equations using graphing, substitution and/or elimination.
M11.D.2.1.5	Solve quadratic equations using factoring.
M11.D.2.2	Simplify expressions involving polynomials.

M11.D.2.2.1	Add, subtract and/or multiply polynomial expressions.
M11.D.2.2.2	Factor algebraic expressions, including difference of squares and trinomials.
M11.D.2.2.3	Simplify algebraic fractions.
M11.D.3	**Analyze change in various contexts.**
M11.D.3.1	Describe and/or determine change.
M11.D.3.1.1	Identify, describe and/or use constant or varying rates of change.
M11.D.3.1.2	Determine how a change in one variable relates to a change in a second variable.
M11.D.3.2	Compute and/or use the slope of a line.
M11.D.3.2.1	Apply the formula for the slope of a line to solve problems.
M11.D.3.2.2	Given the graph of the line, 2 points on the line, or the slope and a point on a line, write or identify the linear equation in point-slope, standard and/or slope-intercept form.
M11.D.3.2.3	Compute the slope and/or y-intercept represented by a linear equation or graph.

NCTM STANDARDS:

Numbers and Operations

- Understand numbers, ways of representing numbers, relationships among numbers, and number systems.
- Understand meanings of operations and how they relate to one another.
- Compute fluently and make reasonable estimates.

Algebra

- Understand patterns, relations, and functions.
- Represent and analyze mathematical situations and structures using algebraic symbols.
- Use mathematical models to represent and understand quantitative relationships.
- Analyze change in various contexts.

Geometry

- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
- Use visualization, spatial reasoning, and geometric modeling to solve problems.

Measurement

- Understand measurable attributes of objects and the units, systems, and processes of measurement.
- Apply appropriate techniques, tools, and formulas to determine measurements.

Problem Solving

- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Monitor and reflect on the process of mathematical problem solving.

UNIT OBJECTIVES:

3.1 Interpret the notion of derivative geometrically. Use it to measure the rate at which things change; and, recognize that derivatives are substitution instances on limit functions.

3.2 Use the basic rules to analytically find derivatives explicitly, implicitly and logarithmically (including derivatives of higher order, derivatives of transcendental functions and their inverse functions).

3.3 Understand and apply statements and theorems about derivatives (i.e. the Mean Value Theorem, L`Hôpital`s Rule and the relationship between continuity and differentiability) to model and solve real-world problems.

3.4 Use technology to support all analytic and numerical facts; and, connect and use previously learned skills and concepts with new analytical methods to confirm data suggested by technology.

Reasoning and Proof • Recognize reasoning and proof as fundamental aspects of mathematics. • Make and investigate mathematical conjectures. • Develop and evaluate mathematical arguments and proofs. • Select and use various types of reasoning and methods of proof. **Communication** • Organize and consolidate their mathematical thinking through communication. • Communicate their mathematical thinking coherently and clearly to peers, teachers, and others. • Analyze and evaluate the mathematical thinking and strategies of others. • Use the language of mathematics to express mathematical ideas precisely. **Connections** • Recognize and use connections among mathematical ideas. • Understand how mathematical ideas interconnect and build on one another to produce a coherent whole. • Recognize and apply mathematics in contexts outside of mathematics. **Representation** • Create and use representations to organize, record, and communicate mathematical ideas. • Select, apply, and translate among mathematical representations to solve problems. • Use representations to model and interpret physical, social, and mathematical phenomena.	

ACTIVITIES:

Constructively participate in individual and cooperative group work to explore, investigate, conjecture, discover, read, write, paraphrase, visualize, verbalize and complete mathematical tasks.

Seek help from the instructor and other facilitators whenever necessary.

Model real world phenomena.

Determine derivatives presented graphically, numerically and analytically.

Interpret the derivative as instantaneous rate of change.

Define the derivative as the limit of the difference quotient.

Understand the relationship between differentiability and continuity.

Determine scopes of secant lines and tangent lines graphically and analytically and determine their relationship to the value of the derivative of a point.

Determine the slope of a curve at a point.

Determine the tangent line to a curve at a point and use it as a linear approximation.

View the Instantaneous rate of change as the limit of the average rate of change.

Approximate rate of change from graphs and tables of values.

Use the definition of derivative to analytically find the derivative function as a limit (equivalent forms).

Investigate one sided derivatives and differentiability of a function at a point.

Use the graphing utility and symmetric difference form of derivative to calculate the value of the derivative at a point.

Derive and apply the differentiation rules.

Determine average and instantaneous velocity and acceleration and other rates of change.

Determine the derivatives of composite functions via application of the Chain Rule.

Use implicit differentiation to find derivations of the union of functions and fractional powers of differentiable functions.

ASSESSMENTS:

- Observation and questioning
- Discussions
- Projects and investigations
- Mathematical writing
- Homework
- Quizzes and Tests

REMEDIATION:

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Use derivative formulas of exponential, logarithmic, trigonometric and inverse trigonometric functions. Use logarithmic differentiation. Determine and apply the relationships that exists between differentiability and continuity. Investigate the Mean Value Theorem and its geometric consequences. Apply the Mean Value Theorem and Rolle's Theorem. Apply L'Hôpital's Rule to determine limits. Use differential formulas to find linear approximations. Determine the derivatives of parametric, polar, and vector functions.	**RESOURCES:** Calculus: Graphical, Numerical, Algebraic 2007 Finney, Demana, Waits, Kennedy Barton, Brunsting, Diehl, Hill, Tyler, Wilson Pearson Education AP Test Prep Series: AP Calculus (student workbook for Finney text) Pearson Education. Boston, Massachusetts, 2007. TI-84 Plus Instructional Manual

Revised 8/08

COURSE: AP Calculus BC	GRADE: 12
UNIT 4: Applications of Derivatives	TIME FRAME: 17 Days

PA ACADEMIC STANDARDS FOR MATHEMATICS:

M11.A.1	**Demonstrate an understanding of numbers, ways of representing numbers, relationships among numbers and number systems.**
M11.A.1.1	Represent and/or use numbers in equivalent forms (e.g., integers, fractions, decimals, percents, square roots, exponents and scientific notation).
M11.A.1.1.1	Find the square root of an integer to the nearest tenth using either a calculator or estimation.
M11.A.1.1.2	Express numbers and/or simplify expressions using scientific notation (including numbers less than 1).
M11.A.1.1.3	Simplify square roots.
M11.A.2	**Understand the meanings of operations, use operations and understand how they relate to each other.**
M11.A.2.1	Apply ratio and/or proportion in problem-solving situations.
M11.A.2.1.1	Solve problems using operations with rational numbers including rates and percents (single and multi-step and multiple procedure operations) (e.g., distance, work and mixture problems, etc.).
M11.A.2.1.2	Solve problems using direct and inverse proportions.
M11.A.2.1.3	Identify and/or use proportional relationships in problem solving settings.
M11.A.2.2	Use exponents, roots and/or absolute value to solve problems.
M11.A.2.2.1	Simplify/evaluate expressions involving positive and negative exponents, roots and/or absolute value (may contain all types of real numbers - exponents should not exceed power of 10).
M11.A.2.2.2	Simplify/evaluate expressions involving multiplying with exponents (e.g. $x^6 * x^7 = x^{13}$), powers of powers (e.g., $(x^6)^7=x^{42}$) and powers of products $(2x^2)^3=8x^6$.
M11.B.2	**Apply appropriate techniques, tools and formulas to determine measurements.**
M11.B.2.1	Use and/or compare measurements of angles.
M11.B.2.1.1	Measure and/or compare angles in degrees (up to 360°).
M11.C.1	**Analyze characteristics and properties of two- and three- dimensional geometric shapes and demonstrate understanding of geometric relationships.**
M11.C.1.4	Solve problems involving right triangles using the Pythagorean Theorem.
M11.C.1.4.1	Find the measure of a side of a right triangle using the Pythagorean Theorem.
M11.D.1	**Demonstrate an understanding of patterns, relations and functions.**
M11.D.1.1	Analyze and/or use patterns or relations.
M11.D.1.1.1	Analyze a set of data for the existence of a pattern and represent the pattern algebraically and/or graphically.
M11.D.1.1.2	Determine if a relation is a function given a set of points or a graph.
M11.D.1.1.3	Identify the domain, range or inverse of a relation (may be presented as ordered pairs or a table).
M11.D.2	**Represent and/or analyze mathematical situations using numbers, symbols, words, tables and/or graphs.**
M11.D.2.1	Write, solve and/or graph linear equations and inequalities using various methods.
M11.D.2.1.1	Solve compound inequalities and/or graph their solution sets on a number line.
M11.D.2.1.2	Identify or graph functions, linear equations or linear inequalities on a coordinate plane.
M11.D.2.1.3	Write, solve and/or apply a linear equation (including problem situations).
M11.D.2.1.4	Write and/or solve systems of equations using graphing, substitution and/or elimination.
M11.D.2.1.5	Solve quadratic equations using factoring.
M11.D.2.2	Simplify expressions involving polynomials.

M11.D.2.2.1	Add, subtract and/or multiply polynomial expressions.
M11.D.2.2.2	Factor algebraic expressions, including difference of squares and trinomials.
M11.D.2.2.3	Simplify algebraic fractions.
M11.D.3	**Analyze change in various contexts.**
M11.D.3.1	Describe and/or determine change.
M11.D.3.1.1	Identify, describe and/or use constant or varying rates of change.
M11.D.3.1.2	Determine how a change in one variable relates to a change in a second variable.
M11.D.3.2	Compute and/or use the slope of a line.
M11.D.3.2.1	Apply the formula for the slope of a line to solve problems.
M11.D.3.2.2	Given the graph of the line, 2 points on the line, or the slope and a point on a line, write or identify the linear equation in point-slope, standard and/or slope-intercept form.
M11.D.3.2.3	Compute the slope and/or y-intercept represented by a linear equation or graph.

NCTM STANDARDS:	UNIT OBJECTIVES:
Numbers and Operations • Understand numbers, ways of representing numbers, relationships among numbers, and number systems. • Understand meanings of operations and how they relate to one another. • Compute fluently and make reasonable estimates. **Algebra** • Understand patterns, relations, and functions. • Represent and analyze mathematical situations and structures using algebraic symbols. • Use mathematical models to represent and understand quantitative relationships. • Analyze change in various contexts. **Geometry** • Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships. • Use visualization, spatial reasoning, and geometric modeling to solve problems. **Measurement** • Understand measurable attributes of objects and the units, systems, and processes of measurement. • Apply appropriate techniques, tools, and formulas to determine measurements. **Problem Solving** • Build new mathematical knowledge through problem solving. • Solve problems that arise in mathematics and in other contexts. • Apply and adapt a variety of appropriate strategies to solve problems. • Monitor and reflect on the process of mathematical problem solving. **Reasoning and Proof** • Recognize reasoning and proof as fundamental aspects of mathematics. • Make and investigate mathematical conjectures. • Develop and evaluate mathematical arguments and proofs. • Select and use various types of reasoning and methods of proof. **Communication** • Organize and consolidate their mathematical thinking through communication. • Communicate their mathematical thinking coherently and clearly to peers, teachers, and others. • Analyze and evaluate the mathematical thinking and strategies of others. • Use the language of mathematics to express mathematical ideas precisely. **Connections** • Recognize and use connections among mathematical	4.1 Use the derivative to analyze and confirm the completeness of a graph that is determined technologically. Predict behavior that is hidden from view or a computer graph. 4.2 Determine and use corresponding characteristics between the graph of a function and its derivatives and vise-versa. 4.3 Model real-world applications of optimization problems and rate-of-change problems. 4.4 Use technology to support all analytical and numerical facts. Use analytical methods to confirm data suggested by technology.

ideas. • Understand how mathematical ideas interconnect and build on one another to produce a coherent whole. • Recognize and apply mathematics in contexts outside of mathematics. **Representation** • Create and use representations to organize, record, and communicate mathematical ideas. • Select, apply, and translate among mathematical representations to solve problems. • Use representations to model and interpret physical, social, and mathematical phenomena.	

ACTIVITIES:	**ASSESSMENTS:**
Constructively participate in individual and cooperative group work to explore, investigate, conjecture, discover, read, write, paraphrase, visualize, verbalize, and complete mathematical tasks. Seek help from the instructor and other facilitators whenever necessary. Use the derivative to find the slope of a curve at a point, tangent and normal lines, critical points, local maximum and minimum values, inflection points and intervals on which the graph is rising, falling, concave up and concave down. Use Newton's method to approximate zeros of a function plus determine differentials and linear approximations of a function. [OPTIONAL] Predict behavior that is hidden from view or a computer graph and confirm the completeness of the graph. Find vertical, horizontal and slant asymptotes. Find extreme values of a function. Use and apply the Mean Value Theorem to theoretical and real-world phenomena. Model real-world applications of average and instantaneous rates of change. Model real-world applications of velocity and acceleration in linear motion. Model related rates-of-change applications. Model real-world optimization problems. Determine corresponding characteristics between the graph of a function and its derivative and vise-versa.	• Observation and questioning • Discussions • Projects and investigations • Mathematical writing • Homework • Quizzes and Tests **REMEDIATION:** *Precalculus Mathematics: A Graphing Approach* , 2nd ed., Demana et. al. Resource Manual *Precalculus Mathematics: A Graphing Approach* , 2nd ed., Demana et. al. Graphing calculator Manual Calculus from Graphical, Numerical, and Symbolic Points of View. Ostebee and Zorn. Lecture Guide and Student Notes Teaching AP Calculus. McMullin, Lin. D&S Marketing Systems. 2005. Logarithm Combination Rules College Board Special Focus: Fundamental Theorem of Calculus. 2006. Describing Vectors (BC unit 9 only) Adding Vectors Graphically (BC unit 9 only) Vector Addition and Scalar Multiplication (BC unit 9 only)

Use the relationship between the increasing and decreasing behavior of f' and the sign of f. Investigate the corresponding characteristics of f, f', and f''. Analyze planar curves given in parametric form, polar form, and vector form, including velocity and acceleration vectors. Geometric interpretation of differential equations via slopefields and the relationship between slopefields and derivatives of implicitly defined functions. Determine numerical solutions of differential equations using Euler's method. Use L'Hôpital's Rule and its application in determining convergence of improper integrals and series.	Rectangular to Polar Components, Standard Position and Norm (BC unit 9 only) **ENRICHMENT:** AP Resources www.apcentral.collegeboard.com Master the AP Calculus AB and BC Test. Kelley, Michael. Thomson Learning. 2003 Mathematics Calculus AB. Brook, Donald. REA. 1995 Mathematics Calculus BC. Brook, Donald. REA. 1995 Multiple Choice and Free Response Questions in Preparation for the AP Calculus Examination. Lederman, David. D&S Marketing Systems. 1998 **RESOURCES:** Calculus: Graphical, Numerical, Algebraic 2007 Finney, Demana, Waits, Kennedy Barton, Brunsting, Diehl, Hill, Tyler, Wilson Pearson Education AP Test Prep Series: AP Calculus (student workbook for Finney text) Pearson Education. Boston, Massachusetts, 2007. TI-84 Plus Instructional Manual

Revised 8/08

COURSE: AP Calculus BC	GRADE: 12
UNIT 5: Integrals	TIME FRAME: 28 Days

PA ACADEMIC STANDARDS FOR MATHEMATICS:

M11.A.1	**Demonstrate an understanding of numbers, ways of representing numbers, relationships among numbers and number systems.**
M11.A.1.1	Represent and/or use numbers in equivalent forms (e.g., integers, fractions, decimals, percents, square roots, exponents and scientific notation).
M11.A.1.1.1	Find the square root of an integer to the nearest tenth using either a calculator or estimation.
M11.A.1.1.2	Express numbers and/or simplify expressions using scientific notation (including numbers less than 1).
M11.A.1.1.3	Simplify square roots.
M11.A.2	**Understand the meanings of operations, use operations and understand how they relate to each other.**
M11.A.2.1	Apply ratio and/or proportion in problem-solving situations.
M11.A.2.1.1	Solve problems using operations with rational numbers including rates and percents (single and multi-step and multiple procedure operations) (e.g., distance, work and mixture problems, etc.).
M11.A.2.1.2	Solve problems using direct and inverse proportions.
M11.A.2.1.3	Identify and/or use proportional relationships in problem solving settings.
M11.A.2.2	Use exponents, roots and/or absolute value to solve problems.
M11.A.2.2.1	Simplify/evaluate expressions involving positive and negative exponents, roots and/or absolute value (may contain all types of real numbers - exponents should not exceed power of 10).
M11.A.2.2.2	Simplify/evaluate expressions involving multiplying with exponents (e.g. $x^6 * x^7 = x^{13}$), powers of powers (e.g., $(x^6)^7=x^{42}$) and powers of products $(2x^2)^3=8x^6$.
M11.B.2	**Apply appropriate techniques, tools and formulas to determine measurements.**
M11.B.2.1	Use and/or compare measurements of angles.
M11.B.2.1.1	Measure and/or compare angles in degrees (up to 360°).
M11.C.1	**Analyze characteristics and properties of two- and three- dimensional geometric shapes and demonstrate understanding of geometric relationships.**
M11.C.1.4	Solve problems involving right triangles using the Pythagorean Theorem.
M11.C.1.4.1	Find the measure of a side of a right triangle using the Pythagorean Theorem.
M11.D.1	**Demonstrate an understanding of patterns, relations and functions.**
M11.D.1.1	Analyze and/or use patterns or relations.
M11.D.1.1.1	Analyze a set of data for the existence of a pattern and represent the pattern algebraically and/or graphically.
M11.D.1.1.2	Determine if a relation is a function given a set of points or a graph.
M11.D.1.1.3	Identify the domain, range or inverse of a relation (may be presented as ordered pairs or a table).
M11.D.2	**Represent and/or analyze mathematical situations using numbers, symbols, words, tables and/or graphs.**
M11.D.2.1	Write, solve and/or graph linear equations and inequalities using various methods.
M11.D.2.1.1	Solve compound inequalities and/or graph their solution sets on a number line.
M11.D.2.1.2	Identify or graph functions, linear equations or linear inequalities on a coordinate plane.
M11.D.2.1.3	Write, solve and/or apply a linear equation (including problem situations).
M11.D.2.1.4	Write and/or solve systems of equations using graphing, substitution and/or elimination.
M11.D.2.1.5	Solve quadratic equations using factoring.
M11.D.2.2	Simplify expressions involving polynomials.

M11.D.2.2.1	Add, subtract and/or multiply polynomial expressions.
M11.D.2.2.2	Factor algebraic expressions, including difference of squares and trinomials.
M11.D.2.2.3	Simplify algebraic fractions.
M11.D.3	**Analyze change in various contexts.**
M11.D.3.1	Describe and/or determine change.
M11.D.3.1.1	Identify, describe and/or use constant or varying rates of change.
M11.D.3.1.2	Determine how a change in one variable relates to a change in a second variable.
M11.D.3.2	Compute and/or use the slope of a line.
M11.D.3.2.1	Apply the formula for the slope of a line to solve problems.
M11.D.3.2.2	Given the graph of the line, 2 points on the line, or the slope and a point on a line, write or identify the linear equation in point-slope, standard and/or slope-intercept form.
M11.D.3.2.3	Compute the slope and/or y-intercept represented by a linear equation or graph.

NCTM STANDARDS:	**UNIT OBJECTIVES:**
Numbers and Operations • Understand numbers, ways of representing numbers, relationships among numbers, and number systems. • Understand meanings of operations and how they relate to one another. • Compute fluently and make reasonable estimates. **Algebra** • Understand patterns, relations, and functions. • Represent and analyze mathematical situations and structures using algebraic symbols. • Use mathematical models to represent and understand quantitative relationships. • Analyze change in various contexts. **Geometry** • Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships. • Use visualization, spatial reasoning, and geometric modeling to solve problems. **Measurement** • Understand measurable attributes of objects and the units, systems, and processes of measurement. • Apply appropriate techniques, tools, and formulas to determine measurements. **Problem Solving** • Build new mathematical knowledge through problem solving. • Solve problems that arise in mathematics and in other contexts. • Apply and adapt a variety of appropriate strategies to solve problems. • Monitor and reflect on the process of mathematical	5.1 Interpret anti-differentiation as the inverse operation of differentiation and use basic integration techniques (including formulas, substitution and integration by parts) to analytically determine anti-derivatives (indefinite integrals). 5.2 Geometrically interpret the definite integral as the area under the graph of a positive function and analytically interpret it as a limit of sums. 5.3 Use the basic formulas and the Fundamental Theorems of Calculus to analytically evaluate definite integrals. Use numerical methods to approximate definite integrals when analytic methods fail or are impractical. 5.4 Use current technology to support analytical and numerical findings. Use analytical methods (including previously learned skills and concepts) to confirm data suggested by technology.

problem solving. **Reasoning and Proof** • Recognize reasoning and proof as fundamental aspects of mathematics. • Make and investigate mathematical conjectures. • Develop and evaluate mathematical arguments and proofs. • Select and use various types of reasoning and methods of proof. **Communication** • Organize and consolidate their mathematical thinking through communication. • Communicate their mathematical thinking coherently and clearly to peers, teachers, and others. • Analyze and evaluate the mathematical thinking and strategies of others. • Use the language of mathematics to express mathematical ideas precisely. **Connections** • Recognize and use connections among mathematical ideas. • Understand how mathematical ideas interconnect and build on one another to produce a coherent whole. • Recognize and apply mathematics in contexts outside of mathematics. **Representation** • Create and use representations to organize, record, and communicate mathematical ideas. • Select, apply, and translate among mathematical representations to solve problems. • Use representations to model and interpret physical, social, and mathematical phenomena.	

ACTIVITIES:	**ASSESSMENTS:**
Constructively participate in individual and cooperative group work to explore, investigate, conjecture, discover, read, write, paraphrase, visualize, verbalize and complete mathematical tasks. Seek help from the instructor and other facilitators whenever necessary. Use the formulas and rules for antiderivatives (indefinite integrals). Use the integration by substitution method. Use the integration by parts method. Use the formulas, rules, and methods to antidifferentiate polynomial, rational, radical, exponential, logarithmic, and trigonometric functions. Integrate parametric, polar, and vector valued functions. Use the rectangle approximation methods (RAM) to estimate area under a curve. Use the algebraic rules and formulas for finite sums. Express the exact area under a curve as a limit of Riemann sums. Compute Riemann sums using left, right, and midpoint evaluation points. Find the exact area under a curve using RAM Riemann sums. Express the definite integral as a limit of Riemann sums and vise-versa. Use the symbols and terminology of integration. Use the definite integral and its properties to find the area under a curve analytically and technologically. Evaluate discontinuous integrable functions to find area under a curve. Interpret and use the definition of average value and Mean Value Theorem for definite integrals. Discuss, interpret and apply the connections between differential and integral calculus. Discuss interpret and use the Fundamental Theorem of Calculus, (Part 1 and Part 2) to evaluate definite integrals and represent a particular antiderivative. Use technology to graph functions of the form $\int_a^x f(t)dt$ (functions whose explicit forms are unknown).	Homework Quizzes Tests Projects **REMEDIATION:** *Precalculus Mathematics: A Graphing Approach* , 2nd ed., Demana et. al. Resource Manual *Precalculus Mathematics: A Graphing Approach* , 2nd ed., Demana et. al. Graphing calculator Manual Calculus from Graphical, Numerical, and Symbolic Points of View. Ostebee and Zorn. Lecture Guide and Student Notes Teaching AP Calculus. McMullin, Lin. D&S Marketing Systems. 2005. Logarithm Combination Rules College Board Special Focus: Fundamental Theorem of Calculus. 2006. Describing Vectors (BC unit 9 only) Adding Vectors Graphically (BC unit 9 only) Vector Addition and Scalar Multiplication (BC unit 9 only) Rectangular to Polar Components, Standard Position and Norm (BC unit 9 only) **ENRICHMENT:** AP Resources www.apcentral.collegeboard.com Master the AP Calculus AB and BC Test. Kelley, Michael. Thomson Learning. 2003 Mathematics Calculus AB. Brook, Donald. REA. 1995 Mathematics Calculus BC. Brook, Donald. REA. 1995 Multiple Choice and Free Response Questions in Preparation for the AP Calculus Examination. Lederman, David. D&S Marketing Systems. 1998

Evaluate definite integrals analytically and confirm technologically (and, if possible, vise-versa). Use the Trapezoidal Rule to approximate the area under a curve. Use Riemann and trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically and by tables of values. Interpret the definite integral of the rate of change of a quantity over an interval as the change of the quantity over the interval $\int_a^b f'(x)dx = f(b) - f(a)$ Evaluate antiderivatives by substitution of variables (including change of limits for definite integrals), parts, and simple partial fractions (non-repeating linear factors only). Evaluate improper integrals as limits of definite integrals.	**RESOURCES:** Calculus: Graphical, Numerical, Algebraic 2007 Finney, Demana, Waits, Kennedy Barton, Brunsting, Diehl, Hill, Tyler, Wilson Pearson Education AP Test Prep Series: AP Calculus (student workbook for Finney text) Pearson Education. Boston, Massachusetts, 2007. TI-84 Plus Instructional Manual

COURSE: AP Calculus BC	GRADE: 12
UNIT 6: Applications of Definite and Indefinite Integrals	TIME FRAME: 25 Days

PA ACADEMIC STANDARDS FOR MATHEMATICS:

M11.A.1	**Demonstrate an understanding of numbers, ways of representing numbers, relationships among numbers and number systems.**
M11.A.1.1	Represent and/or use numbers in equivalent forms (e.g., integers, fractions, decimals, percents, square roots, exponents and scientific notation).
M11.A.1.1.1	Find the square root of an integer to the nearest tenth using either a calculator or estimation.
M11.A.1.1.2	Express numbers and/or simplify expressions using scientific notation (including numbers less than 1).
M11.A.1.1.3	Simplify square roots.
M11.A.2	**Understand the meanings of operations, use operations and understand how they relate to each other.**
M11.A.2.1	Apply ratio and/or proportion in problem-solving situations.
M11.A.2.1.1	Solve problems using operations with rational numbers including rates and percents (single and multi-step and multiple procedure operations) (e.g., distance, work and mixture problems, etc.).
M11.A.2.1.2	Solve problems using direct and inverse proportions.
M11.A.2.1.3	Identify and/or use proportional relationships in problem solving settings.
M11.A.2.2	Use exponents, roots and/or absolute value to solve problems.
M11.A.2.2.1	Simplify/evaluate expressions involving positive and negative exponents, roots and/or absolute value (may contain all types of real numbers - exponents should not exceed power of 10).
M11.A.2.2.2	Simplify/evaluate expressions involving multiplying with exponents (e.g. $x^6 * x^7 = x^{13}$), powers of powers (e.g., $(x^6)^7=x^{42}$) and powers of products $(2x^2)^3=8x^6$.
M11.B.2	**Apply appropriate techniques, tools and formulas to determine measurements.**
M11.B.2.1	Use and/or compare measurements of angles.
M11.B.2.1.1	Measure and/or compare angles in degrees (up to 360°).
M11.C.1	**Analyze characteristics and properties of two- and three- dimensional geometric shapes and demonstrate understanding of geometric relationships.**
M11.C.1.4	Solve problems involving right triangles using the Pythagorean Theorem.
M11.C.1.4.1	Find the measure of a side of a right triangle using the Pythagorean Theorem.
M11.D.1	**Demonstrate an understanding of patterns, relations and functions.**
M11.D.1.1	Analyze and/or use patterns or relations.
M11.D.1.1.1	Analyze a set of data for the existence of a pattern and represent the pattern algebraically and/or graphically.
M11.D.1.1.2	Determine if a relation is a function given a set of points or a graph.
M11.D.1.1.3	Identify the domain, range or inverse of a relation (may be presented as ordered pairs or a table).
M11.D.2	**Represent and/or analyze mathematical situations using numbers, symbols, words, tables and/or graphs.**
M11.D.2.1	Write, solve and/or graph linear equations and inequalities using various methods.
M11.D.2.1.1	Solve compound inequalities and/or graph their solution sets on a number line.
M11.D.2.1.2	Identify or graph functions, linear equations or linear inequalities on a coordinate plane.
M11.D.2.1.3	Write, solve and/or apply a linear equation (including problem situations).
M11.D.2.1.4	Write and/or solve systems of equations using graphing, substitution and/or elimination
M11.D.2.1.5	Solve quadratic equations using factoring.
M11.D.2.2	Simplify expressions involving polynomials.

M11.D.2.2.1	Add, subtract and/or multiply polynomial expressions.
M11.D.2.2.2	Factor algebraic expressions, including difference of squares and trinomials.
M11.D.2.2.3	Simplify algebraic fractions.
M11.D.3	**Analyze change in various contexts.**
M11.D.3.1	Describe and/or determine change.
M11.D.3.1.1	Identify, describe and/or use constant or varying rates of change.
M11.D.3.1.2	Determine how a change in one variable relates to a change in a second variable.
M11.D.3.2	Compute and/or use the slope of a line.
M11.D.3.2.1	Apply the formula for the slope of a line to solve problems.
M11.D.3.2.2	Given the graph of the line, 2 points on the line, or the slope and a point on a line, write or identify the linear equation in point-slope, standard and/or slope-intercept form.
M11.D.3.2.3	Compute the slope and/or y-intercept represented by a linear equation or graph.

NCTM STANDARDS:

Numbers and Operations
- Understand numbers, ways of representing numbers, relationships among numbers, and number systems.
- Understand meanings of operations and how they relate to one another.
- Compute fluently and make reasonable estimates.

Algebra
- Understand patterns, relations, and functions.
- Represent and analyze mathematical situations and structures using algebraic symbols.
- Use mathematical models to represent and understand quantitative relationships.
- Analyze change in various contexts.

Geometry
- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
- Use visualization, spatial reasoning, and geometric modeling to solve problems.

Measurement
- Understand measurable attributes of objects and the units, systems, and processes of measurement.
- Apply appropriate techniques, tools, and formulas to determine measurements.

Problem Solving
- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Monitor and reflect on the process of mathematical problem solving.

UNIT OBJECTIVES:

6.1 Use antiderivatives to solve separable differential equations and model real-world phenomena involving distance, velocity and acceleration with initial conditions, and growth and decay.

6.2 Find area under and between curves, volumes of solids of revolution, and volumes of solids with known cross sections.

6.3 Model real-world phenomena involving area and volume.

6.4 Use technology to support all analytical and numerical facts and use analytic methods including previously learned skills and concepts, to confirm (whenever possible) data suggested by technology.

Reasoning and Proof • Recognize reasoning and proof as fundamental aspects of mathematics. • Make and investigate mathematical conjectures. • Develop and evaluate mathematical arguments and proofs. • Select and use various types of reasoning and methods of proof. **Communication** • Organize and consolidate their mathematical thinking through communication. • Communicate their mathematical thinking coherently and clearly to peers, teachers, and others. • Analyze and evaluate the mathematical thinking and strategies of others. • Use the language of mathematics to express mathematical ideas precisely. **Connections** • Recognize and use connections among mathematical ideas. • Understand how mathematical ideas interconnect and build on one another to produce a coherent whole. • Recognize and apply mathematics in contexts outside of mathematics. **Representation** • Create and use representations to organize, record, and communicate mathematical ideas. • Select, apply, and translate among mathematical representations to solve problems. • Use representations to model and interpret physical, social, and mathematical phenomena.	

ACTIVITIES:	**ASSESSMENTS:**
Constructively participate in individual and cooperative group work to explore, investigate, conjecture, discover, read, write, paraphrase, visualize, verbalize and complete mathematical tasks. Seek help from the instructor and other facilitators whenever necessary. Solve separable differential equations and real world initial value applications of growth and decay. Find velocity given acceleration and distance given initial values for velocity and acceleration. Find the distance traveled by a particle along a line. Find solution curves. Model other real-world phenomena given initial conditions. Solve logistic differential equations and use them in modeling. Use integrals to determine the length of curves in the plane. Use integrals to model work and fluid pressure problems. Find area between curves by integrating with respect to x and also with respect to y. Determine volumes of solids of revolution using disks and washers. Determine volumes of solids of revolution using cylindrical shells. Determine volumes of solids of revolution generated by overlapping regions. Determine volume of solids with known cross sections using the slicing method. Determine volumes of solids that model real world phenomena. Find the average valve of a function. Find the area of a region bounded by polar curves. Determine the length of a curve including a curve given in parametric or polar form. Use integrals in applications with emphasis on using the integral of a rate of change to give accumulated change or using the method of setting up an approximating Riemann sum and representing its limit	Homework Quizzes Tests Projects **REMEDIATION:** *Precalculus Mathematics: A Graphing Approach* , 2nd ed., Demana et. al. Resource Manual *Precalculus Mathematics: A Graphing Approach* , 2nd ed., Demana et. al. Graphing calculator Manual Calculus from Graphical, Numerical, and Symbolic Points of View. Ostebee and Zorn. Lecture Guide and Student Notes Teaching AP Calculus. McMullin, Lin. D&S Marketing Systems. 2005. Logarithm Combination Rules College Board Special Focus: Fundamental Theorem of Calculus. 2006. Describing Vectors (BC unit 9 only) Adding Vectors Graphically (BC unit 9 only) Vector Addition and Scalar Multiplication (BC unit 9 only) Rectangular to Polar Components, Standard Position and Norm (BC unit 9 only) **ENRICHMENT:** AP Resources www.apcentral.collegeboard.com Master the AP Calculus AB and BC Test. Kelley, Michael. Thomson Learning. 2003 Mathematics Calculus AB. Brook, Donald. REA. 1995 Mathematics Calculus BC. Brook, Donald. REA. 1995 Multiple Choice and Free Response Questions in Preparation for the AP Calculus Examination. Lederman, David. D&S Marketing Systems. 1998

as a definite integral.	**RESOURCES:** Calculus: Graphical, Numerical, Algebraic 2007 Finney, Demana, Waits, Kennedy Barton, Brunsting, Diehl, Hill, Tyler, Wilson Pearson Education AP Test Prep Series: AP Calculus (student workbook for Finney text) Pearson Education. Boston, Massachusetts, 2007. TI-84 Plus Instructional Manual

Revised 8/08

COURSE: AP Calculus BC	GRADE: 12
UNIT 7: Polynomial Approximations and Series	TIME FRAME: 23 Days

PA ACADEMIC STANDARDS FOR MATHEMATICS:

M11.A.1	**Demonstrate an understanding of numbers, ways of representing numbers, relationships among numbers and number systems.**
M11.A.1.1	Represent and/or use numbers in equivalent forms (e.g., integers, fractions, decimals, percents, square roots, exponents and scientific notation).
M11.A.1.1.1	Find the square root of an integer to the nearest tenth using either a calculator or estimation.
M11.A.1.1.2	Express numbers and/or simplify expressions using scientific notation (including numbers less than 1).
M11.A.1.1.3	Simplify square roots.
M11.A.2	**Understand the meanings of operations, use operations and understand how they relate to each other.**
M11.A.2.1	Apply ratio and/or proportion in problem-solving situations.
M11.A.2.1.1	Solve problems using operations with rational numbers including rates and percents (single and multi-step and multiple procedure operations) (e.g., distance, work and mixture problems, etc.).
M11.A.2.1.2	Solve problems using direct and inverse proportions.
M11.A.2.1.3	Identify and/or use proportional relationships in problem solving settings.
M11.A.2.2	Use exponents, roots and/or absolute value to solve problems.
M11.A.2.2.1	Simplify/evaluate expressions involving positive and negative exponents, roots and/or absolute value (may contain all types of real numbers - exponents should not exceed power of 10).
M11.A.2.2.2	Simplify/evaluate expressions involving multiplying with exponents (e.g. $x^6 * x^7 = x^{13}$), powers of powers (e.g., $(x^6)^7=x^{42}$) and powers of products $(2x^2)^3=8x^6$.
M11.B.2	**Apply appropriate techniques, tools and formulas to determine measurements.**
M11.B.2.1	Use and/or compare measurements of angles.
M11.B.2.1.1	Measure and/or compare angles in degrees (up to 360°).
M11.C.1	**Analyze characteristics and properties of two- and three- dimensional geometric shapes and demonstrate understanding of geometric relationships.**
M11.C.1.4	Solve problems involving right triangles using the Pythagorean Theorem.
M11.C.1.4.1	Find the measure of a side of a right triangle using the Pythagorean Theorem.
M11.D.1	**Demonstrate an understanding of patterns, relations and functions.**
M11.D.1.1	Analyze and/or use patterns or relations.
M11.D.1.1.1	Analyze a set of data for the existence of a pattern and represent the pattern algebraically and/or graphically.
M11.D.1.1.2	Determine if a relation is a function given a set of points or a graph.
M11.D.1.1.3	Identify the domain, range or inverse of a relation (may be presented as ordered pairs or a table).
M11.D.2	**Represent and/or analyze mathematical situations using numbers, symbols, words, tables and/or graphs.**
M11.D.2.1	Write, solve and/or graph linear equations and inequalities using various methods.
M11.D.2.1.1	Solve compound inequalities and/or graph their solution sets on a number line.
M11.D.2.1.2	Identify or graph functions, linear equations or linear inequalities on a coordinate plane.
M11.D.2.1.3	Write, solve and/or apply a linear equation (including problem situations).
M11.D.2.1.4	Write and/or solve systems of equations using graphing, substitution and/or elimination.
M11.D.2.1.5	Solve quadratic equations using factoring.
M11.D.2.2	Simplify expressions involving polynomials.

M11.D.2.2.1	Add, subtract and/or multiply polynomial expressions.
M11.D.2.2.2	Factor algebraic expressions, including difference of squares and trinomials.
M11.D.2.2.3	Simplify algebraic fractions.
M11.D.3	**Analyze change in various contexts.**
M11.D.3.1	Describe and/or determine change.
M11.D.3.1.1	Identify, describe and/or use constant or varying rates of change.
M11.D.3.1.2	Determine how a change in one variable relates to a change in a second variable
M11.D.3.2	Compute and/or use the slope of a line.
M11.D.3.2.1	Apply the formula for the slope of a line to solve problems.
M11.D.3.2.2	Given the graph of the line, 2 points on the line, or the slope and a point on a line, write or identify the linear equation in point-slope, standard and/or slope-intercept form.
M11.D.3.2.3	Compute the slope and/or y-intercept represented by a linear equation or graph.

NCTM STANDARDS:	**UNIT OBJECTIVES:**
Numbers and Operations • Understand numbers, ways of representing numbers, relationships among numbers, and number systems. • Understand meanings of operations and how they relate to one another. • Compute fluently and make reasonable estimates. **Algebra** • Understand patterns, relations, and functions. • Represent and analyze mathematical situations and structures using algebraic symbols. • Use mathematical models to represent and understand quantitative relationships. • Analyze change in various contexts. **Geometry** • Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships. • Use visualization, spatial reasoning, and geometric modeling to solve problems. **Measurement** • Understand measurable attributes of objects and the units, systems, and processes of measurement. • Apply appropriate techniques, tools, and formulas to determine measurements. **Problem Solving** • Build new mathematical knowledge through problem solving. • Solve problems that arise in mathematics and in other contexts.	7.1 Define a series of a sequence of partial sums. 7.2 Interpret the convergence of a series as the limit of a sequence of partial sums. 7.3 Investigate the convergence or divergence of a series of constants using numerical, analytical, and graphical techniques. 7.4 Use a power series (or Taylor Series) to approximate functions. 7.5 Use technology to support all analytical and numerical facts. Use analytic methods including previously learned skills and concepts to confirm data suggested by technology.

• Apply and adapt a variety of appropriate strategies to solve problems. • Monitor and reflect on the process of mathematical problem solving. **Reasoning and Proof** • Recognize reasoning and proof as fundamental aspects of mathematics. • Make and investigate mathematical conjectures. • Develop and evaluate mathematical arguments and proofs. • Select and use various types of reasoning and methods of proof. **Communication** • Organize and consolidate their mathematical thinking through communication. • Communicate their mathematical thinking coherently and clearly to peers, teachers, and other. • Analyze and evaluate the mathematical thinking and strategies of others. • Use the language of mathematics to express mathematical ideas precisely. **Connections** • Recognize and use connections among mathematical ideas. • Understand how mathematical ideas interconnect and build on one another to produce a coherent whole. • Recognize and apply mathematics in contexts outside of mathematics. **Representation** • Create and use representations to organize, record, and communicate mathematical ideas. • Select, apply, and translate among mathematical representations to solve problems. • Use representations to model and interpret physical, social, and mathematical phenomena.	

ACTIVITIES:

Constructively participate in individual and cooperative group work to explore, investigate, conjecture, discover, read, write, paraphrase, visualize, verbalize, and complete mathematical tasks.

Seek help from the instructor and other facilitators whenever necessary.

Investigate examples of sequences and series, including decimal expansion.

Determine the convergence or divergence of geometric series with applications, the harmonic series, alternating series with error bound, and p-series using numerical, analytical, and graphical techniques.

Use the integral test, comparison test, ratio test, and root test to determine the convergence or divergence of a series.

Approximate functions using Taylor Polynomials and confirm with a graphical demonstration of convergence.

Determine the Maclaurin series and general Taylor series centered at x = a for a function.

Determine the Maclaurin series for the functions e^x, sin x, cos x, and $\frac{1}{1-x}$.

Use formal manipulation of Taylor series and shortcuts to computing Taylor series, including substitution, differentiation, anti-differentiation, and the formation of new series from known series.

Investigate functions defined by power series.

Determine the radius and interval of convergence of power series.

Determine the Lagrange error bound for Taylor polynomials.

ASSESSMENTS:

Homework
Quizzes
Tests
Projects

REMEDIATION:

Precalculus Mathematics: A Graphing Approach , 2nd ed., Demana et. al. Resource Manual

Precalculus Mathematics: A Graphing Approach , 2nd ed., Demana et. al. Graphing calculator Manual

Calculus from Graphical, Numerical, and Symbolic Points of View. Ostebee and Zorn.

Lecture Guide and Student Notes

Teaching AP Calculus. McMullin, Lin. D&S Marketing Systems. 2005.

Logarithm Combination Rules

College Board Special Focus: Fundamental Theorem of Calculus. 2006.

Describing Vectors (BC unit 9 only)

Adding Vectors Graphically (BC unit 9 only)

Vector Addition and Scalar Multiplication (BC unit 9 only)

Rectangular to Polar Components, Standard Position and Norm (BC unit 9 only)

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ENRICHMENT:

AP Resources
www.apcentral.collegeboard.com

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Mathematics Calculus AB. Brook, Donald. REA. 1995

Mathematics Calculus BC. Brook, Donald. REA. 1995

Multiple Choice and Free Response Questions in Preparation for the AP Calculus Examination. Lederman, David. D&S Marketing Systems. 1998

	RESOURCES: Calculus: Graphical, Numerical, Algebraic 2007 Finney, Demana, Waits, Kennedy Barton, Brunsting, Diehl, Hill, Tyler, Wilson Pearson Education AP Test Prep Series: AP Calculus (student workbook for Finney text) Pearson Education. Boston, Massachusetts, 2007. TI-84 Plus Instructional Manual

Revised 8/08

COURSE: AP Calculus BC	GRADE: 12
UNIT 8: AP Exam Review	TIME FRAME: 25 Days

PA ACADEMIC STANDARDS FOR MATHEMATICS:

M11.A.1	**Demonstrate an understanding of numbers, ways of representing numbers, relationships among numbers and number systems.**
M11.A.1.1	Represent and/or use numbers in equivalent forms (e.g., integers, fractions, decimals, percents, square roots, exponents and scientific notation).
M11.A.1.1.1	Find the square root of an integer to the nearest tenth using either a calculator or estimation.
M11.A.1.1.2	Express numbers and/or simplify expressions using scientific notation (including numbers less than 1).
M11.A.1.1.3	Simplify square roots.
M11.A.2	**Understand the meanings of operations, use operations and understand how they relate to each other.**
M11.A.2.1	Apply ratio and/or proportion in problem-solving situations.
M11.A.2.1.1	Solve problems using operations with rational numbers including rates and percents (single and multi-step and multiple procedure operations) (e.g., distance, work and mixture problems, etc.).
M11.A.2.1.2	Solve problems using direct and inverse proportions.
M11.A.2.1.3	Identify and/or use proportional relationships in problem solving settings.
M11.A.2.2	Use exponents, roots and/or absolute value to solve problems.
M11.A.2.2.1	Simplify/evaluate expressions involving positive and negative exponents, roots and/or absolute value (may contain all types of real numbers - exponents should not exceed power of 10).
M11.A.2.2.2	Simplify/evaluate expressions involving multiplying with exponents (e.g. $x^6 * x^7 = x^{13}$), powers of powers (e.g., $(x^6)^7=x^{42}$) and powers of products $(2x^2)^3=8x^6$.
M11.B.2	**Apply appropriate techniques, tools and formulas to determine measurements.**
M11.B.2.1	Use and/or compare measurements of angles.
M11.B.2.1.1	Measure and/or compare angles in degrees (up to 360°).
M11.C.1	**Analyze characteristics and properties of two- and three- dimensional geometric shapes and demonstrate understanding of geometric relationships.**
M11.C.1.4	Solve problems involving right triangles using the Pythagorean Theorem.
M11.C.1.4.1	Find the measure of a side of a right triangle using the Pythagorean Theorem.
M11.D.1	**Demonstrate an understanding of patterns, relations and functions.**
M11.D.1.1	Analyze and/or use patterns or relations.
M11.D.1.1.1	Analyze a set of data for the existence of a pattern and represent the pattern algebraically and/or graphically.
M11.D.1.1.2	Determine if a relation is a function given a set of points or a graph.
M11.D.1.1.3	Identify the domain, range or inverse of a relation (may be presented as ordered pairs or a table).
M11.D.2	**Represent and/or analyze mathematical situations using numbers, symbols, words, tables and/or graphs.**
M11.D.2.1	Write, solve and/or graph linear equations and inequalities using various methods.
M11.D.2.1.1	Solve compound inequalities and/or graph their solution sets on a number line.
M11.D.2.1.2	Identify or graph functions, linear equations or linear inequalities on a coordinate plane.
M11.D.2.1.3	Write, solve and/or apply a linear equation (including problem situations).
M11.D.2.1.4	Write and/or solve systems of equations using graphing, substitution and/or elimination.
M11.D.2.1.5	Solve quadratic equations using factoring.
M11.D.2.2	Simplify expressions involving polynomials.

M11.D.2.2.1	Add, subtract and/or multiply polynomial expressions.
M11.D.2.2.2	Factor algebraic expressions, including difference of squares and trinomials.
M11.D.2.2.3	Simplify algebraic fractions.
M11.D.3	**Analyze change in various contexts.**
M11.D.3.1	Describe and/or determine change.
M11.D.3.1.1	Identify, describe and/or use constant or varying rates of change.
M11.D.3.1.2	Determine how a change in one variable relates to a change in a second variable.
M11.D.3.2	Compute and/or use the slope of a line.
M11.D.3.2.1	Apply the formula for the slope of a line to solve problems.
M11.D.3.2.2	Given the graph of the line, 2 points on the line, or the slope and a point on a line, write or identify the linear equation in point-slope, standard and/or slope-intercept form.
M11.D.3.2.3	Compute the slope and/or y-intercept represented by a linear equation or graph.

NCTM STANDARDS:

Numbers and Operations

- Understand numbers, ways of representing numbers, relationships among numbers, and number systems.
- Understand meanings of operations and how they relate to one another.
- Compute fluently and make reasonable estimates.

Algebra

- Understand patterns, relations, and functions.
- Represent and analyze mathematical situations and structures using algebraic symbols.
- Use mathematical models to represent and understand quantitative relationships.
- Analyze change in various contexts.

Geometry

- Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
- Use visualization, spatial reasoning, and geometric modeling to solve problems.

Measurement

- Understand measurable attributes of objects and the units, systems, and processes of measurement.
- Apply appropriate techniques, tools, and formulas to determine measurements.

Problem Solving

- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Monitor and reflect on the process of mathematical

UNIT OBJECTIVES:

8.1	Work collaboratively in order to strengthen verbal and written mathematical communication and construction of arguments.
8.2	Work independently in order to evaluate their personal strengths and weaknesses.
8.3	Use graphing calculators to support conclusions and solve problems.
8.4	Discuss and practice test taking tips and strategies.
8.5	Commit formulas and theorems to memory.

problem solving. **Reasoning and Proof** • Recognize reasoning and proof as fundamental aspects of mathematics. • Make and investigate mathematical conjectures. • Develop and evaluate mathematical arguments and proofs. • Select and use various types of reasoning and methods of proof. **Communication** • Organize and consolidate their mathematical thinking through communication. • Communicate their mathematical thinking coherently and clearly to peers, teachers, and others. • Analyze and evaluate the mathematical thinking and strategies of others. • Use the language of mathematics to express mathematical ideas precisely. **Connections** • Recognize and use connections among mathematical ideas. • Understand how mathematical ideas interconnect and build on one another to produce a coherent whole. • Recognize and apply mathematics in contexts outside of mathematics. **Representation** • Create and use representations to organize, record, and communicate mathematical ideas. • Select, apply, and translate among mathematical representations to solve problems. • Use representations to model and interpret physical, social, and mathematical phenomena.	

ACTIVITIES:

Review activities are coordinated by the teacher using published resources and past AP Exams released by the College Board. Students will work cooperatively and independently to review all course material in preparation for the AP Exam.

RESOURCES:

Calculus: Graphical, Numerical, Algebraic 2007
Finney, Demana, Waits, Kennedy

Barton, Brunsting, Diehl, Hill, Tyler, Wilson
Pearson Education AP Test Prep Series: AP Calculus (student workbook for Finney text)
Pearson Education. Boston, Massachusetts, 2007.

TI-84 Plus Instructional Manual

AP Resources
www.apcentral.collegeboard.com

Practice Books:

Lederman, David
Multiple Choice & Free Response Questions in Preparation for the AP Calculus BC Examination (Sixth Edition)
D&S Marketing Systems, Inc. Brooklyn, New York, 1998.

Arterburn, Hubbard, Perl
The Best Test Preparation for the Advanced Placement Examination: Mathematics Calculus BC
Research and Education Association. Piscataway, NJ, 1997.

ASSESSMENTS:
Homework
Quizzes
Practice Exams
Discussion

REMEDIATION:

Precalculus Mathematics: A Graphing Approach , 2nd ed., Demana et. al. Resource Manual

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ENRICHMENT:

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COURSE: AP Calculus BC	GRADE: 12
UNIT 9: Topics in Calculus	TIME FRAME: 25 Days

PA ACADEMIC STANDARDS FOR MATHEMATICS:

M11.A.1	**Demonstrate an understanding of numbers, ways of representing numbers, relationships among numbers and number systems.**
M11.A.1.1	Represent and/or use numbers in equivalent forms (e.g., integers, fractions, decimals, percents, square roots, exponents and scientific notation).
M11.A.1.1.1	the square root of an integer to the nearest tenth using either a calculator or estimation.
M11.A.1.1.2	numbers and/or simplify expressions using scientific notation (including numbers less than 1).
M11.A.1.1.3	Simplify square roots.
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M11.D.2.1.4	Write and/or solve systems of equations using graphing, substitution and/or elimination.
M11.D.2.1.5	Solve quadratic equations using factoring.

M11.D.2.2	Simplify expressions involving polynomials.
M11.D.2.2.1	Add, subtract and/or multiply polynomial expressions.
M11.D.2.2.2	Factor algebraic expressions, including difference of squares and trinomials.
M11.D.2.2.3	Simplify algebraic fractions.
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M11.D.3.1	Describe and/or determine change.
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M11.D.3.1.2	Determine how a change in one variable relates to a change in a second variable.
M11.D.3.2	Compute and/or use the slope of a line.
M11.D.3.2.1	Apply the formula for the slope of a line to solve problems.
M11.D.3.2.2	Given the graph of the line, 2 points on the line, or the slope and a point on a line, write or identify the linear equation in point-slope, standard and/or slope-intercept form.
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NCTM STANDARDS:

Numbers and Operations
- Understand numbers, ways of representing numbers, relationships among numbers, and number systems.
- Understand meanings of operations and how they relate to one another.
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Algebra
- Understand patterns, relations, and functions.
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- Apply appropriate techniques, tools, and formulas to determine measurements.

Problem Solving
- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Monitor and reflect on the process of mathematical

UNIT OBJECTIVES:

9.1 Use previously learned mathematical skills and concepts to establish connections to new material.

9.2 Use technology as a tool to investigate mathematical concepts and ideas, to provide support for analytical work and to solve problems using approximation when analytic methods either fail or are impractical.

problem solving. **Reasoning and Proof** • Recognize reasoning and proof as fundamental aspects of mathematics. • Make and investigate mathematical conjectures. • Develop and evaluate mathematical arguments and proofs. • Select and use various types of reasoning and methods of proof. **Communication** • Organize and consolidate their mathematical thinking through communication. • Communicate their mathematical thinking coherently and clearly to peers, teachers, and others. • Analyze and evaluate the mathematical thinking and strategies of others. • Use the language of mathematics to express mathematical ideas precisely. **Connections** • Recognize and use connections among mathematical ideas. • Understand how mathematical ideas interconnect and build on one another to produce a coherent whole. • Recognize and apply mathematics in contexts outside of mathematics. **Representation** • Create and use representations to organize, record, and communicate mathematical ideas. • Select, apply, and translate among mathematical representations to solve problems. • Use representations to model and interpret physical, social, and mathematical phenomena.	

ACTIVITIES:

Activities will vary depending on teacher's preference. Topics covered should be calculus-based and connected to the course concepts. Suggestions include, but are not limited to:

Functions of Three Variables

Analysis of Three-dimensional surfaces

Partial Derivatives

Vectors in Three Space

Cylindrical Coordinates

Spherical Coordinates

ASSESSMENTS:

Homework
Quizzes
Tests
Projects

REMEDIATION:

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Vector Addition and Scalar Multiplication (BC unit 9 only)

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ENRICHMENT:

AP Resources
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Multiple Choice and Free Response Questions in Preparation for the AP Calculus Examination. Lederman, David. D&S Marketing Systems. 1998

	RESOURCES: Calculus: Graphical, Numerical, Algebraic 2007 Finney, Demana, Waits, Kennedy Barton, Brunsting, Diehl, Hill, Tyler, Wilson Pearson Education AP Test Prep Series: AP Calculus (student workbook for Finney text) Pearson Education. Boston, Massachusetts, 2007. TI-84 Plus Instructional Manual WinPlot software Larson, Hostetler, Edwards Calculus (Sixth edition) Houghton Mifflin Company. Boston, Massachusetts, 1998.

Revised 8/08

AP Calculus BC Course Description

The AP Calculus BC course is a one semester class meeting for 90 days, 90 minutes per day. Students who enroll in BC in the spring have taken and passed AP Calculus AB during the fall semester, and generally go on to take the AP Calculus BC Exam. Conceptual understanding is emphasized, and there is an essential balance in the course between technology and pencil and paper calculations. Each student has his or her own TI-84 plus calculator issued by the school to use for the duration of the course.

Through lecture, independent exploration, practice and collaboration, the students gain an in-depth understanding of the material and make valuable connections between and among topics. At the beginning of each class, the students work together on an introductory problem or activity, discussing ideas and approaches to reach a solution. Students are encouraged to ask questions and engage in class discussions. Homework is collected weekly and graded for completeness and correctness. Students are expected to complete out of class assignments and may collaborate on them at the beginning of the next class, allowing students the opportunity to communicate mathematics verbally on a daily basis.

Note: The following outline contains both semesters of AP Calculus. Calculus BC topics that are covered in the first semester are denoted by an asterisk (*). Calculus BC topics that are covered in the second semester are denoted by a double asterisk (**).

Course Outline

I. Unit 1: Functions and Graphs (Precalculus Review) 6 Days
   a. Linear Functions
   b. Exponential Functions
   c. Logarithmic Functions
   d. Inverse Functions
   e. Parametric Curves
   f. Trigonometric Functions

Teaching Notes:

The focus is on the analysis of graphs and characteristics of functions. Local and global behavior of functions is considered numerically, analytically, and by utilizing the support of graphing technology. Students are expected to make connections between the various representations. Also, the importance of considering all of the representations is emphasized. For example, students are expected to realize that graphical representation is not always sufficient to describe characteristics of functions (e.g. removable discontinuity). From the very beginning of the course, students are encouraged to formulate mathematically sound arguments and reasonable solutions in both verbal and written form.

II. Unit 2: Limits and Continuity (10 Days)
   a. Limits
      i. Informal definition of Limits and Notation
      ii. Properties of Limits
      iii. Formal Definition of Limit
      iv. Limits Involving Infinity
      v. Asymptotic Behavior
   b. Definition of Continuity
      i. Properties of continuity
      ii. Continuous extension
      iii. Intermediate Value Theorem
   c. Tangent Lines
      i. Average Rate of Change
      ii. Instantaneous Rate of Change
      iii. Tangents and Normal lines

Teaching Notes:

Limits are introduced using a graphical, numerical and analytical approach. One-sided limits are also evaluated and explored. Early on, the concept of infinity is incorporated through end behavior and asymptotic behavior of graphs. A strong emphasis is placed on reasons why a limit fails to exist. Graphical exploration is followed by analytical interpretation. Students use the Squeeze (Sandwich) Theorem to solidify their

understanding of limits and begin their work with proofs. Verbal or written explanation is required along with their proof.

Continuity is initially approached from a graphical perspective. The idea of proof is revisited with the Intermediate Value Theorem. Through the understanding of local linearity and limits, students construct the concept of the tangent line.

III. Derivatives (14 Days)
- a. Definition of Derivative
- b. Differentiability
- c. Introduction to the relationship between the graphs of f and f'
- d. Differentiation Rules
- e. Rates of Change
- f. Derivatives of Trigonometric Functions
- g. Chain Rule
- h. Parametric Differentiation
- i. Implicit Differentiation
- j. Derivatives of Exponential and Logarithmic Functions
- k. Derivatives of Inverse Trigonometric Functions

Teaching Notes:

In the beginning of the unit, major emphasis is placed on the relationship between differentiability and continuity. Connections are made between the derivative, tangent line, slope of curve, average rate of change, and instantaneous rate of change. A major emphasis is placed on local linearity, requiring students to discuss the concept in their own words. Graphical exploration is utilized to "zoom-in" on various graphs to determine differentiability. Once conceptual understanding has been accomplished, the focus is shifted to computing derivatives and rules and techniques for differentiation. The differentiation rules are initially explored through the use of graphing technology. Students make conjectures about derivatives of polynomial, trigonometric and exponential functions. After exploration, many of the derivatives of trigonometric, exponential, logarithmic and inverse trigonometric functions are derived by the students using proofs. In particular, proofs of inverse trigonometric functions are written and verbally presented to the class by student teams.

IV. Applications of the Derivative (15 days)
- a. Mean and Extreme Value Theorem
- b. Increasing and Decreasing intervals
- c. Concavity
- d. Connecting f with f' and f''
- e. Modeling and Optimization
- f. Linearization
- g. Differentials
- h. Related Rates

Teaching Notes:

This unit is built on two major concepts. In the beginning of the unit, the emphasis is for students to make connections between a function and its first and second derivatives. This is done graphically, algebraically, and numerically. Also, the concept of the derivative as a rate of change is reinforced with reference to position, velocity and acceleration.

The second part of the unit focuses on the application of the derivative to solving problems, such as optimization and related rates problems. Students solve problems analytically, supporting their conclusions with the use of graphing technology. Students gain an appreciation for the usefulness of calculus through real-world scenarios. Also, the concept of approximating functions using tangent lines and differentials is explored graphically, numerically and analytically.

V. Integrals (14 Days)
- a. Riemann Sums
  - i. Right
  - ii. Left
  - iii. Midpoint
- b. Definite Integrals
- c. Fundamental Theorem of Calculus
- d. Trapezoidal Approximation of Definite Integrals
- e. Indefinite Integrals
- f. Integration Techniques

i. Integration by Substitution*
ii. Integration by Parts and Tabular Integration*

Teaching Notes:

During this unit, there is a strong emphasis of the concept of a definite integral as a numerical value. Approximations involving Riemann Sums and Trapezoids are explored and compared. Right, Left, and Midpoint Riemann Sums and Trapezoidal approximations are done by hand and using programs for the graphing calculator. Students look at indefinite integrals as mathematical objects, mainly general antiderivatives. Integration properties, formulas and techniques are introduced and practiced. In particular, integration by substitution is carried out both by changing the limits of integration in the substitution and also returning to the original expression and using the given limits of integration. Relationships between functions and their antideriva tives are explored using graphing technology, particularly the relationship between position, velocity, and acceleration. One of the major concepts of this unit is the Fundamental Theorem of Calculus, Parts 1 and 2. Students are exposed to a variety of problems involving the FTC and its applications. Students are given excerpts from the College Board's AP Calculus 2005-2006 Professional Development Workshop Materials Special Focus: The Fundamental Theorem of Calculus. Students work collaboratively to reach a deep level of understanding of the FTC, teaching students how to communicate mathematics verbally. Released AB Free Response questions are used to facilitate proper written explanation of mathematical concepts.

VI. Applications of Definite and Indefinite Integrals (22 days)
a. Differential Equations
b. Slope Fields
c. Separable Differential Equations
d. Law of Exponential Change
e. Euler's Method*
f. Partial Fraction Decomposition*
g. Integration by Partial Fractions*
h. Logistic Differential equations*
i. Integrals as Net Change
j. Areas in the Plane
i. Area under curve
ii. Area between curves
k. Lengths of Curves (including parametric)*
l. Volumes
i. Disc method
ii. Washer method
iii. Shell method
iv. Cross-sectional area

Teaching Notes:

In the first part of this unit, the focus is on differential equations, using slope fields as a method to find solution curves and Euler's method as a numerical approximation of the function's value. Students draw slope fields by hand and use a slope field program for the graphing calculators. Discussion is facilitated by use of the overhead projection capabilities of the graphing calculator. Calculations for Euler's method are done by hand using step-by-step iteration and using a calculator program. Students develop written and verbal arguments for whether the solution produced by Euler's method is an over- or under-estimate of the actual solution based on both analytical and graphical analysis. After students have obtained a solid understanding of the concept of a differential equation, separable differential equations are introduced and exponential growth and decay are emphasized.

The second part of the unit focuses on applications of integrals as an accumulation of a rate of change, distance, area and volume. Formulas for lengths of curves are introduced. Students make geometric connections between curve lengths of circles in function and parametric form using circumference. As an introduction to volume, computer-based solids of revolutions are explored at http://college.hmco.com/mathematics/larson/calculus_analytic/7e/students/3dgraphs/ch06.html. Volume is calculated using the disc, washer and shell methods, and using a variety of known cross-sectional areas.

VII. Improper Integrals** (10 days)
a. L'Hopital's Rule and Indeterminate Form
b. Relative Growth Rates
c. Improper Integrals
i. Infinite limits of integration

ii. Discontinuities
iii. Applications
iv. Comparison Test

Teaching Notes:

In the beginning of the unit, students explore limits of functions having indeterminate forms graphically and numerically. The concept of indeterminate form is discussed. Limits are calculated analytically using L'Hopital's Rule. Improper integrals are explored as limits of definite integrals. Several functions with horizontal asymptotes or infinite discontinuities are integrated analytically and students are encouraged to discuss the concept of integrating to infinity. The method of partial fractions and L'Hopital's Rule is used in the evaluation of improper integrals, when necessary. The Comparison test for convergence or divergence of improper integrals is used, with support from graphing technology. Solids with infinite surface area and finite volume (such as 'Gabriel's Horn') are used to enhance the application of improper integrals.

VIII. Polynomial Approximations and Series** (25 days)
- a. Sequences
  - i. Explicitly defined
  - ii. Recursively defined
  - iii. Arithmetic
  - iv. Geometric
  - v. Graphing
- b. Series
- c. Power Series
- d. Taylor and Maclaurin Series
  - i. Manipulating Taylor and Maclaurin series as algebraic objects
  - ii. Approximating polynomials using series
  - iii. Error (including Lagrange Error Bound)
- e. Convergence Tests
  - i. Radius and Interval of Convergence
  - ii. n^{th} term test
  - iii. comparison test
  - iv. integral test
  - v. ratio test
  - vi. alternating series test
  - vii. p-series

Teaching Notes:

This unit starts out by taking a look at sequences. Students use formulas to find terms and write explicit and recursive formulas from a list of terms. In particular, arithmetic and geometric sequences are studied. Graphing calculators are used to graph sequences in parametric mode (for explicit formulas) and in sequence mode (for recursive formulas).

After an understanding of sequences is developed, the concept of a series is introduced. Students construct a sequence of partial sums to determine convergence or divergence of the series. Convergence is determined numerically, graphically or analytically using a formula for the n^{th} partial sum. Students express repeating decimals as an infinite series. The study of series continues with power series. In particular, geometric power series and their convergence or divergence is explored. Graphing calculators support the idea of representing a function with a power series and the interval of convergence. Power series are manipulated as algebraic objects, using term-by-term differentiation and integration. Again, conclusions are supported graphically.

In the next part of the unit, polynomial approximation using Taylor and Maclaurin series is emphasized. Students use the general formula for writing a Taylor series and explore the approximation numerically and graphically. Error is calculated analytically and graphically, and by using the Lagrange error bound. Students become very familiar with the Taylor series for $\sin x$, $\cos x$, and e^x. They perform manipulations on Taylor series, including differentiation, integration, substitution, addition, and subtraction.

The last part of the unit emphasizes convergence of series. The radius and interval of convergence are determined analytically and supported graphically. The discussion begins with the n^{th} term test and geometric, harmonic, alternating, and p-series. The ratio test is used to expand the idea to other power series. Convergence is also determined using integral and comparison tests. Throughout this part of the unit, students are encouraged

to work collaboratively to construct their arguments for convergence or divergence. They express arguments verbally and in writing.

IX. Parametric, Polar and Vector Functions** (12 days)
- a. Parametric Equations
- b. Polar Coordinates
- c. Slope of Polar Curves
- d. Area of Polar Region
- e. Vectors in the Plane
- f. Calculus of Vectors
  - i. Derivatives of Vector Functions
  - ii. Position, Velocity, Acceleration
  - iii. Speed
  - iv. Displacement
  - v. Distance Traveled

Teaching Notes:

Characteristics of Parametric, Polar and Vector functions are studied and analyzed. In particular, students explore the different representations of circles and lines in all forms to make comparisons. Graphing technology is used to enhance the study of parametric curves, modeling direction of motion and paths of particles. Students explore particle motion using simultaneous graphing capabilities and analytic analysis.

Polar coordinates are introduced for basic understanding and some graphing is done by hand. Students explore different types of polar curves through a graphing calculator activity involving cardiods, limacons, rose curves, etc…. They work collaboratively to make connections between the equation and the type of graph that is produced, communicating mathematics verbally and in writing. After a basic understanding of polar functions is established, derivatives and integrals of polar functions are studied. Slope of a polar curve and area of a polar region is considered. Emphasis is placed on limits of integration.

Vector functions are introduced with an emphasis on application and connections to parametric functions. Navigation, position, velocity, acceleration, and speed problems are explored. Derivatives and integrals of vector functions are considered using application problems to motivate the teaching. Students make connections between distance traveled by a particle in vector form and the formula for curve length of a parametric curve.

X. AP Exam Review (25 days)
- a. Workbooks
- b. Free Response
- c. Released Exams
- d. Practice Exams
- e. Tips and Strategies

Teaching Notes:

Following the completion of the curriculum, students are involved in an extensive review for the AP Calculus BC exam. Throughout the review, students work collaboratively in order to strengthen their verbal and written mathematical communication and construction of arguments. They also work independently in order to evaluate their personal strengths and weaknesses. Graphing calculators are used to support conclusions and solve problems. Test taking tips and strategies are shared, discussed and practiced. Published practice books and released College Board materials are utilized during the review. Formulas and theorems are reviewed and committed to memory.

Assessments

Semester One (AB):

Exam: Chapter 1
Quiz: Section 2.1, 2.2
Exam: Chapter 2
Quiz: Sections 3.5 – 3.9
Exam: Chapter 3
Quiz: Section 4.1 – 4.4
Quiz: Section 4.5, 4.6
Quiz: Definite Integrals
Exam: Chapter 5
Quiz: Section 6.2, 6.3
Quiz: Section 6.1, 6.4
Exam: Chapter 6
Quiz: Section 7.1, 7.2, 7.4
Exam: Chapter 7

Exam: Chapter 4 Final Exam

Semester Two (BC):

Exam: Chapter 8 AP Free Response Questions
Exam: Chapter 9 Released Exams
Exam: Chapter 10 Formula and Theorem Quizzes (3)

In addition to teacher-generated quizzes and exams, students are also exposed to released Free-response questions from past AB exams and multiple choice practice problems written for AP Exam preparation. These problems are chosen and/or written to facilitate written student explanation of concepts.

Primary textbook

Finney, Demana, Waits, Kennedy
Calculus: Graphical, Numerical, Algebraic (Third Edition) AP Edition
Pearson Prentice Hall. Boston, Massachusetts, 2007.

Supplemental Resources

Textbooks and Workbooks:

1. Barton, Brunsting, Diehl, Hill, Tyler, Wilson
Pearson Education AP Test Prep Series: AP Calculus (student workbook for Finney text)
Pearson Education. Boston, Massachusetts, 2007.

2. Larson, Hostetler, Edwards
Calculus (Sixth edition)
Houghton Mifflin Company. Boston, Massachusetts, 1998.

3. Lederman, David
Multiple Choice & Free Response Questions in Preparation for the AP Calculus BC Examination (Sixth Edition)
D&S Marketing Systems, Inc. Brooklyn, New York, 1998.

4. Arterburn, Hubbard, Perl
The Best Test Preparation for the Advanced Placement Examination: Mathematics Calculus BC
Research and Education Association. Piscataway, NJ, 1997.

Websites:

1. AP Central

http://apcentral.collegeboard.com/apc/Controller.jpf

2. Houghton Mifflin: Calculus with Analytic Geometry

http://college.hmco.com/mathematics/larson/calculus_analytic/7e/students/3dgraphs/ch06.html.